Why We Have Never Used the Black-Scholes-Merton Option Pricing Formula

Espen Gaarder Haug & Nassim Nicholas Taleb

September 2007- Very Preliminary Draft – Cannot be Quoted or Disseminated

Abstract: Options traders use a pricing formula which they adapt by fudging and changing the tails and skewness by varying one parameter, the standard deviation of a Gaussian. Such formula is popularly called "Black-Scholes-Merton" owing to an attributed eponymous discovery (though changing the standard deviation parameter is in contradiction with it). However we have historical evidence that 1) Black, Scholes and Merton did not invent any formula, just found an argument to make a well known (and used) formula compatible with the economics establishment, by removing the "risk" parameter through "dynamic hedging", 2) Option traders use (and evidently have used since 1902) the previous versions of the formula of Louis Bachelier and Edward O. Thorp (that allow a broad choice of probability distributions) and removed the risk parameter by using put-call parity. The Bachelier-Thorp approach is more robust (among other things) to the high impact rare event. It is time to stop calling the formula by the wrong name.

BREAKING THE CHAIN OF TRANSMISSION

Tricks and heuristically derived methodologies in option trading and risk management of derivatives books have been developed over the past century, and used quite effectively by operators. In parallel, many derivations were produced by mathematical researchers. The economics literature, however, did not recognize these contributions, substituting the rediscoveries or subsequent reformulations done by (some) economists. There is evidence of an attribution problem with Black-Scholes-Merton option "formula", which was developed, used, and adapted in a robust way by a long tradition of researchers and used heuristically by operators. Furthermore, in a case of scientific puzzle, the exact formula called "Black-Sholes-Merton" was written down (and used) by Edward Thorp which, paradoxically, while being robust and realistic, was considered unrigorous. This raises the following: 1) The Black Scholes Merton was just a neoclassical finance argument, 2) Traders neither use their argument nor their version of the formula.

It is high time to give credit where it belongs.

THE BLACK-SCHOLES-MERTON "FORMULA" WAS AN ARGUMENT

Option traders call the formula they use the "Black-Scholes-Merton" formula without being aware that by some irony, of all the possible options formulas that have been produced in the past century, what is called the Black-Scholes-Merton "formula" (after Black and Scholes, 1973, and Merton, 1973) is the one the furthest away from what they are using. In fact of the formulas written down in a long history it is the only formula that was fragile to jumps and tail events.

First, something seems to have been lost in translation: Black and Scholes (1973) and Merton (1973) actually never came up with a *new* option formula, but only an theoretical economic *argument* built on a new way of "deriving", rather rederiving, an already existing –and well known –formula. The argument, we will see, is extremely fragile to assumptions. The foundations of option hedging and pricing were already far more firmly laid down before them. Their argument, simply, is that an option can be hedged using a certain methodology called "dynamic hedging" and then turned into a risk-free instrument, as the portfolio would no longer be stochastic. Indeed what Black Scholes and Merton did was "marketing", finding a way to make a well-known formula palatable to the economics establishment of the time, little else, and in fact distorting its essence.

Such argument requires strange far-fetched assumptions: some liquidity at the level of transactions, knowledge of the probabilities of future events (in a neoclassical Arrow-Debreu style), and, more critically, a certain mathematical structure that requires "thin-tails", or mild randomness, on which, later. The entire argument is indeed, quite strange and rather inapplicable for someone clinically and observation-driven standing outside conventional neoclassical economics. Simply, the dynamic hedging argument is dangerous in practice as it subjects you to blowups; it makes no sense unless you are concerned with neoclassical economic theory. The Black-Scholes-Merton argument and equation flow a top-down general equilibrium theory, built upon the assumptions of operators working *in full knowledge* of the probability distribution of future outcomes –in addition to a collection of assumptions that, we will see, are highly invalid mathematically, the main one being the ability to cut the risks using continuous trading which only works in the very narrowly special case of thin-tailed distributions. But it is not just these flaws that make it inapplicable: option traders do not "buy theories", particularly speculative general equilibrium ones, which they find too risky for them and extremely lacking in standards of reliability. A normative theory is, simply, not good for decision-making under uncertainty (particularly if it is in chronic disagreement with empirical evidence). People may take decisions based on speculative theories, but avoid the fragility of theories in running their risks.

Yet professional traders, including the authors (and, alas, the Nobel committee) have operated under the illusion that it was the Black-Scholes-Merton "formula" they actually used –we were told so. This myth has been progressively reinforced in the literature and in business schools, as the original sources have been lost or frowned upon as "anecdotal" (Merton, 1992).

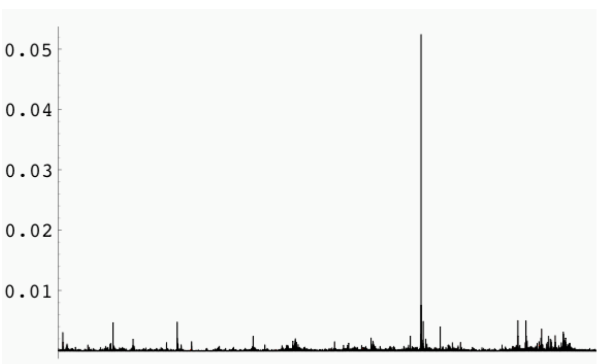

Figure 1 The typical "risk reduction" performed by the Black-Scholes-Merton argument. These are the variations of a dynamically hedged portfolio. BSM indeed "smoothes" out risks but exposes the operator to massive tail events –reminiscent of such blowups as LTCM. Other option formulas are robust to the rare event and make no such claims.

This discussion will present our real-world, ecological understanding of option pricing and hedging based on what option traders actually do and did for more than a hundred years.

This is a very general problem. Option traders develop a chain of transmission of knowledge, like many professions. But the problem is that the "chain" is often broken as universities do not store the acquired skills by operators. Effectively plenty of robust heuristically derived implementations have been developed over the years, but the economics establishment has refused to quote them or acknowledge them. This makes traders need to relearn matters periodically. Failure of dynamic hedging in 1987, by such firm as Leland O'Brien Rubinstein, for instance, does not seem to appear in the academic literature published after the event (Merton, 1992, Rubinstein, 1998, Ross, 2005); to the contrary dynamic hedging is held to be a standard operation[12].

There are central elements of the real world that can escape them –academic research without feedback from practice (in a *practical* and applied field) can cause the diversions we witness between laboratory and ecological frameworks. This explains why some many finance academics have had the tendency to make smooth returns, then blow up using their own theories[3]. We started the other way around, first by years of option trading doing million of hedges and thousands of option trades. This in combination with investigating the forgotten and ignored ancient knowledge in option pricing and trading we will explain some common myths about option pricing and hedging.

There are indeed two myths:

- That we had to wait for the Black-Scholes-Merton options formula to trade the product, price options, and manage option books. In fact the introduction of the Black, Scholes and Merton argument increased our risks and set us back in risk management.
- That we "use" the Black-Scholes-Merton options "pricing formula". We, simply don't.

In our discussion of these myth we will focus on the bottom-up literature on option theory that has been hidden in the dark recesses of libraries. And that addresses only recorded matters –not the actual practice of option trading that has been lost.

1 For instance –how mistakes never resurface into the consciousness, Mark Rubinstein was awarded in 1995 the Financial Engineer of the Year award by the International Association of Financial Engineers. There was no mention of portfolio insurance and the failure of dynamic hedging.

3 For a standard reaction to a rare event, see this Wall Street Journal article:
"Wednesday is the type of day people will remember in quant-land for a very long time," said Mr. Rothman, a University of Chicago Ph.D. who ran a quantitative fund before joining Lehman Brothers. "Events that models only predicted would happen once in 10,000 years happened every day for three days." One 'Quant' Sees Shakeout For the Ages -- '10,000 Years' By Kaja Whitehouse, August 11, 2007; Page B3.

MYTH 1: PEOPLE DID NOT PROPERLY "PRICE" OPTIONS BEFORE THE BLACK-SCHOLES-MERTON THEORY

It is assumed that the Black-Scholes-Merton theory is what made it possible for option traders to calculate their delta hedge (against the underlying) and to price options. This argument is highly debatable, both historically and analytically.

Options were actively trading at least already in the 1600 as described by Joseph De La Vega –implying some form of heuristic method to price them and deal with their exposure. De La Vega describes option trading in the Netherlands, indicating that operators were of some expertise in option theory. For example he diffusely points to the put-call parity, and his book was not even meant to teach people about the technicalities in option trading. Our insistence on the use of Put-Call parity is critical for the following reason: The Black-Scholes-Merton's claim to fame is removing the necessity of a risk-based drift from the underlying security –to make the trade "risk-neutral". But one does not need dynamic hedging for that: simple put call parity can suffice (Derman and Taleb, 2005), as we will discuss later. And it is this central removal of the "risk-premium" that apparently was behind the decision by the Nobel committee to grant Merton and Scholes the Bank of Sweden Prize in honor of Alfred Nobel: "Black, Merton and Scholes made a vital contribution by showing that it is in fact not necessary to use any risk premium when valuing an option. This does not mean that the risk premium disappears; instead it is already included in the stock price."[4] It is for having removed the effect of the drift on the value of the option that their work was originally cited, something that was mechanically present by any form of trading and converting using far simpler techniques.

In the late 1800 and the early 1900 there were active option markets in London and New York as well as in Paris and several other European exchanges. There was even active option arbitrage trading going on between several of these markets.

In 1904, an option trader and arbitrageur, S.A. Nelson published a book "The A B C of Options and Arbitrage". According to Nelson (1904) up to 500 messages per hour and typically 2000 to 3000 messages per day where sent between the London and the New York market through the cable companies. Each message was transmitted over the wire system in less than a minute. In a heuristic method that was repeated in *Dynamic Hedging* by one of the authors, (Taleb,1997), Nelson, describe in a theory-free way many rigorously clinical aspects of his arbitrage business: the cost of shipping shares, the cost of insuring shares, interest expenses, the possibilities to switch shares directly between someone being long securities in New York and short in London and in this way saving shipping and insurance costs, as well as many more tricks etc.

For example the put-call parity was according to modern option literature first fully described by Stoll (1969), but they never even mention Nelson. The fact is that the put-call parity argument was fully understood and described in detail already by Nelson (1904) which in turn makes frequent reference to Higgins (1902). Just as an example Nelson (1904) referring to Higgins (1902) writes:

> "It may be worthy of remark that 'calls' are more often dealt than 'puts' the reason probably being that the majority of 'punters' in stocks and shares are more inclined to look at the bright side of things, and therefore more often 'see' a rise than a fall in prices.
>
> This special inclination to buy 'calls' and to leave the 'puts' severely alone does not, however, tend to make 'calls' dear and 'puts' cheap, for it can be shown that the adroit dealer in options can convert a 'put' into a 'call,' a 'call' into a 'put', a 'call o' more' into a 'put- and-call,' in fact any option into another, by dealing against it in the stock. We may therefore assume, with tolerable accuracy, that the 'call' of a stock at any moment costs the same as the 'put' of that stock, and half as much as the Put-and-Call."

The Put-and-Call was simply a put plus a call with the same strike and maturity, what we today would call a straddle. Nelson describes the put-call parity over many pages in full detail. Static market neutral delta hedging was also known at that time, in his book Nelson for example writes:

> "Sellers of options in London as a result of long experience, if they sell a Call, straightway buy half the stock against which the Call is sold; or if a Put is sold; they sell half the stock immediately."

We must see this in the light that standard options in London at that time were issued at-the-money as explicitly pointed out by Nelson; furthermore, all standard options in London were European style. In London in- or out-of-the-money options where only traded occasionally and where known as "fancy options". It is quite clear from this and the rest of Nelson's book that that the option dealers where well aware of the delta for at-the-money options was approximately 50%. As a matter of fact at-the-money options trading in London at that time were adjusted to be struck to be at-the-money forward, in order to make puts and calls of the same price. We know today know that options that are at-the-money forward and not have very long time to maturity have a delta very close to 50% (naturally minus 50% for puts). The options in London at that time typically had one month to maturity when issued.

Nelson also diffusely points to dynamic delta hedging, and that it worked better in theory than practice (Haug, 2007). It is clearly from all the details described by Nelson that options in the early 1900 traded actively

[4] see www.Nobel.se

and that option traders at that time in no way felt helpless in either pricing or in hedging them. Herbert Filer was another option trader that was involved in option trading from 1919 to the 1960s. Filler(1959) describes what must be consider a reasonable active option market in New York and Europe in the early 1920s and 1930s. Filer mention however that due to World War II there was no trading on the European Exchanges, for they were closed. Further, he mentions that London option trading did not resume before 1958. In the early 1900, option traders in London were considered to be the most sophisticated, according to Nelson. It could well be that World War II and the subsequent shutdown of option trading for many years was the reason known robust arbitrage principles about options were forgotten and almost lost, to be partly re-discovered by finance professors such as Stoll (1969).

In 1908 Vinzenz Bronzin published a book deriving several option pricing formulas, and a formula very similar to what today is known as the Black-Scholes-Merton formula. Bronzin based his *risk-neutral* option valuation on robust arbitrage principles such as the put-call parity and the link between the forward price and call and put options –in a way that was rediscovered by Derman and Taleb (2005). Indeed, the put-call parity restriction is sufficient to remove the need to incorporate a future return in the underlying security –it forces the lining up of options to the forward price[5].

Again, 1910 Henry Deutsch describes put-call parity but in less detail than Higgins and Nelson. In 1961 Reinach again described the put-call parity in quite some detail (another text typically ignored by academics). Traders at New York stock exchange specializing in using the put-call parity to convert puts into calls or calls into puts was at that time known as Converters. Reinach (1961):

> "Although I have no figures to substantiate my claim, I estimate that over 60 per cent of all Calls are made possible by the existence of Converters."

In other words the *converters* (dealers) who basically operated as market makers were able to operate and hedge most of their risk by "statically" hedging options with options. Reinach wrote that he was an option trader (Converter) and gave examples on how he and his colleagues tended to hedge and arbitrage options against options by taking advantage of options embedded in convertible bonds:

> "Writers and traders have figured out other procedures for making profits writing Puts & Calls. Most are too specialized for all but the seasoned professional. One such procedure is the ownership of a convertible bonds and then writing of Calls against the stock into which the bonds are convertible. If the stock is called converted and the stock is delivered."

Higgins, Nelson and Reinach all describe the great importance of the put-call parity and *to hedge options with options*. Option traders where in no way helpless in hedging or pricing before the Black-Scholes-Merton formula. Based on simple arbitrage principles they where able to hedge options more robustly than with Black- Scholes-Merton. As already mentioned static market-neutral delta hedging was described by Higgins and Nelson in 1902 and 1904. Also, W. D. Gann (1937) discusses market neutral delta hedging for at-the-money options, but in much less details than Nelson (1904). Gann also indicates some forms of auxiliary dynamic hedging.

Option Formulas and Delta Hedging

What about option pricing formulas. Did we have any option pricing formulas before Black-Scholes-Merton 1973. The first identifiable one was Bachelier (1900). Sprenkle (1962) extended Bacheliers work to assume lognormal rather than normal distributed asset price. James Boness (1964) also assumed a lognormal asset price.

Boness derives a formula for the price of a call option that is actually identical to the Black-Scholes-Merton 1973 formula, but the way Black, Scholes and Merton derived their formula based on continuous dynamic delta hedging or alternatively based on CAPM they were able to get independent of the expected rate of return. It is in other words not the formula itself that is considered the great discovery done by Black, Scholes and Merton, but how they derived it. This is among several others also pointed out by Rubinstein (2006):

> "The real significance of the formula to the financial theory of investment lies not in itself, but rather in how it was derived. Ten years earlier the same formula had been derived by Case M. Sprenkle (1962) and A. James Boness (1964)."

[5] Ruffino and Treussard (2006) accept that one could have solved the risk-premium by happenstance, not realizing that put-call parity was so extensively used in history. But they find it insufficient. Indeed the argument may not be *sufficient* for someone who subsequently complicated the representation of the world with some implements of modern finance such as "stochastic discount rates" –while simplifying it at the same time to make it limited to the Gaussian and allowing dynamic hedging. They write that "the use of a non-stochastic discount rate common to both the call and the put options is inconsistent with modern equilibrium capital asset pricing theory." Given that the authors have never seen a practitioner use "stochastic discount rate", we, like our option trading predecessors, feel that put-call parity is sufficient & does the job.

The situation is akin to that of scientists lecturing birds on how to fly, and taking credit for their subsequent performance –except that here it would be lecturing them the wrong way.

Samuelson 1969 and also Thorp 1969 published somewhat similar option pricing formulas to Boness and Sprenkle. Thorp (2007) claims that he actually had an identical formula to the Black-Scholes-Merton formula programmed into his computer years before Black, Scholes and Merton published their theory.

Now, delta hedging. As already mentioned static market-neutral delta hedging was described by Higgins and Nelson 1902 and 1904. Thorp and Kassouf (1967) described market neutral static delta hedging in more details, not only for at-the-money options, but for options *with any delta*. In his 1969 paper Thorp is shortly describing market neutral static delta hedging, also briefly pointed in the direction of some dynamic delta hedging, not as a central pricing device, but a risk-management tool. Filer also points to dynamic hedging of options, but without showing much knowledge about how to calculate the delta. Another "ignored" and "forgotten" text is a book/booklet published in 1970 by Arnold Bernhard & Co. The authors are clearly aware of market neutral static delta hedging or what they name "balanced hedge" for any level in the strike or asset price. This book has multiple examples of how to buy warrants or convertible bonds and construct a market neutral delta hedge by shorting the right amount of common shares. Arnold Bernhard & Co also published deltas for a large number of warrants and convertible bonds that they distributed to investors on Wall Street.

Referring to Thorp and Kassouf (1967), Black, Scholes and Merton took the idea of delta hedging one step further, Black and Scholes (1973):

> "If the hedge is maintained continuously, then the approximations mentioned above become exact, and the return on the hedged position is completely independent of the change in the value of the stock. In fact, the return on the hedged position becomes certain. This was pointed out to us by Robert Merton."

This may be a brilliant mathematical idea, but option trading is not mathematical theory. It is not enough to have a theoretical idea so far removed from reality that is far from robust in practice. What is surprising is that the only principle option traders do not use and cannot use is the approach named after the formula, which is a point we discuss next.

MYTH 2: OPTION TRADERS TODAY "USE" THE BLACK-SCHOLES-MERTON FORMULA

Traders don't do "Valuation"

First, operationally, a price is not quite "valuation". Valuation requires a strong theoretical framework with its corresponding fragility to both assumptions and the structure of a model. For traders, a "price" produced to buy an option when one has no knowledge of the probability distribution of the future is not "valuation", but an expedient. Such price could change. Their beliefs do not enter such price. It can also be determined by his inventory.

This distinction is critical: traders are engineers, whether boundedly rational (or even non interested in any form of probabilistic rationality), they are not privy to informational transparency about the future states of the world and their probabilities. So they do not need a general theory to produce a price –merely the avoidance of Dutch-book style arbitrages against them, and the compatibility with some standard restriction: In addition to put-call parity, a call of a certain strike K cannot trade at a lower price than a call K+ΔK (avoidance of negative call and put spreads), a call struck at K and a call struck at K+2 ΔK cannot be more expensive that twice the price of a call struck at K+ΔK (negative butterflies), horizontal calendar spreads cannot be negative (when interest rates are low), and so forth.

In that sense, traders do not perform "valuation" with some "pricing kernel" until the expiration of the security, but, rather, produce a price of an option compatible with other instruments in the markets, with a holding time that is stochastic. They do not need top-down "science".

On the Mathematical Impossibility of Dynamic Hedging

Finally, we discuss the severe flaw in the dynamic hedging concept. It assumes, nay, requires all moments of the probability distribution to exist[6].

Assume that the distribution of returns has a scale-free or fractal property that we can simplify as follows: for x large enough, (i.e. "in the tails"), P[X>n x]/P[X>x] depends on n, not on x. In financial securities, say, where X is a daily return, there is no reason for P[X>20%]/P[X>10%] to be different from P[X>15%]/P[X>7.5%]. This self-similarity at all scales generates power-law, or Paretian, tails, i.e., above a crossover point, $P[X>x]=K\ x^{-\alpha}$. It happens, looking at millions of pieces of data, that such property holds in markets –all markets, baring sample error. For overwhelming empirical evidence, see Mandelbrot (1963), which predates Black-Scholes-Merton (1973) and the jump-diffusion or Merton (1976), Stanley et al (2000), and Gabaix et al (2003).

Some criticism of fat-tails accept that such property might apply for daily returns, but, owing to the Central Limit Theorem, should not work for weekly ones as the distribution becomes Gaussian under aggregation.

[6] Merton (1992) seemed to accept the inapplicability of dynamic hedging but he perhaps thought that these ills would be cured thanks to his prediction of the financial world "spiraling towards dynamic completeness". Fifteen years later, if anything, we may have spiraled *away* from it.

However the argument does not hold owing to the preasymptotics of scalable distributions: Bouchaud and Potters (2003) and Mandelbrot and Taleb (2007) show that the presasymptotics of fractal distributions are such that the effect of the Central Limit Theorem are exceedingly slow in the tails –in fact irrelevant. Furthermore, there is sampling error as we have less data for longer periods, hence fewer tail episodes, which give an in-sample illusion of thinner tails. In addition, the point that aggregation thins out the tails does not hold for dynamic hedging –in which the operator depends necessarily on high frequency data and their statistical properties. So long as it is scale-free at the time period of dynamic hedge, higher moments become explosive, "infinite" to disallow the formation of a dynamically hedge portfolio. Simply a Taylor explansion is impossible as moments of higher order that 2 matter critically.

The mechanics of dynamic hedging are as follows. Assume the risk-free interest rate of 0 with no loss of generality. The canonical Black-Scholes-Merton package consists in selling a call and purchasing shares of stock that provide a hedge against instantaneous moves in the security. Thus the portfolio π locally "hedged" against exposure to the first moment of the distribution is the following:

$$\pi = -C + \frac{\partial C}{\partial S} S$$

where C is the call price, and S the underlying security.

Take the change in the values of the portfolio

$$\Delta\pi = -\Delta C + \frac{\partial C}{\partial S} \Delta S$$

By expanding around the initial values of S, we have the changes in the portfolio in discrete time. Conventional option theory applies to the Gaussian in which all orders higher than ΔS^2 and disappears rapidly.

$$\Delta\pi = -\frac{\partial C}{\partial t} \Delta t - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \Delta S^2 + O(\Delta S^3)$$

Taking expectations on both sides, we can see here very strict requirements on moment finiteness: all moments need to converge. If we include another term, of order ΔS^3, such term may be of significance in a probability distribution with significant cubic or quartic terms. Indeed, although the n^{th} derivative with respect to S can decline very sharply, for options that have a strike K away from the center of the distribution, it remains that the moments are rising disproportionately fast for that to carry a mitigating effect.

So here we mean *all* moments--no approximation. The logic of the Black-Scholes-Merton so-called solution thanks to Ito's lemma was that the portfolio collapses into a deterministic payoff. But let us see how quickly or effectively this works in practice.

The Actual Replication process is as follows: The payoff of a call should be replicated with the following stream of dynamic hedges, the limit of which can be seen here, between t and T

$$\lim_{\Delta t \to 0} \left(\sum_{i=1}^{n=\frac{T}{\Delta t}} \frac{\partial C}{\partial S} \Bigg|_{S=S_{t+(i-1)\Delta t},\, t=t+(i-1)\Delta t,} \left(S_{t+i\Delta t} - S_{t+(i-1)\Delta t}\right) \right)$$

Such policy does not match the call value: the difference remains stochastic. Unless one lives in a fantasy world in which such risk reduction is possible.

Further, there is an inconsistency in the works of Merton making us confused as to what theory finds acceptable: in Merton (1976) he agrees that we can use Bachelier-style option derivation in the presence of jumps and discontinuities, but only when the underlying stock price is uncorrelated to the market. This seems to be an admission that the dynamic hedging argument applies only to *some* securities: those that do not jump and are correlated to the market.

The Robustness of the Gaussian

The success of the "formula" last developed by Thorp, and called "Black-Scholes-Merton" was due to a simple attribute of the Gaussian: you can express *any* probability distribution in terms of Gaussian, even if it has fat tails, by varying the standard deviation σ at the level of the density of the random variable. It does not mean that you are using a Gaussian, nor does it mean that the Gaussian is particularly parsimonious (since you have to attach a σ for every level of the price). It simply mean that the Gaussian can express anything you want if you add a function for the parameter σ, making it function of strike price and time to expiration.

This "volatility smile" , i.e., varying one parameter to produce $\sigma(K)$, or "volatility surface", varying two parameter, $\sigma(S,t)$ is effectively what was done in different ways by Derman (1994,1998), Dupire(1994, 2005), see Gatheral(2006). They assume a volatility process not because there is necessarily such a thing – only as a method of fitting option prices to a Gaussian. Furthermore, although the Gaussian has finite second moment (and finite all higher moments as well), you can express a scalable with infinite variance using Gaussian "volatility surface". One strong constrain on the σ parameter is that it must be the same for a put and call with same strike (if both are European-style), and the drift should be that of the forward.

Indeed, ironically, the volatility smile is inconsistent with the Black-Scholes-Merton theory. This has lead to hundreds if not thousands of papers trying extend (what was perceived to be) the Black-Scholes-Merton model to incorporate stochastic volatility and jump-diffusion. Several of these researchers are surprised that so few traders actually use stochastic volatility models. It is not a model that says how the volatility smile should look like, or evolves over time; it is a hedging method that is robust and consistent with an arbitrage free volatility surface that evolves over time.

In other words, you can use a volatility surface as a map, not a territory. However it is foolish to justify Black-Scholes-Merton on grounds of its use: it bans the use of probability distributions that are not Gaussian – whereas non-dynamic hedging derivations (Bachelier, Thorp) are not grounded in the Gaussian.

Order Flow and Options

It is clear that option traders are not necessarily interested in probability distribution at expiration time – given that this is abstract, even metaphysical for them. In addition to the put-call parity constrains that according to evidence was fully developed already in 1904, we can hedge away inventory risk in options with other options. One very important implication of this method is that if you hedge options with options then option pricing will be largely demand and supply based[7]. This in strong contrast to the Black-Scholes-Merton (1973) theory that based on the idealized world of geometric Brownian motion with continuous-time delta hedging then demand and supply for options simply not should affect the price of options. If someone wants to buy more options the market makers can simply manufacture them by dynamic delta hedging that will be a perfect substitute for the option itself.

This raises a critical point: option traders do not "estimate" the odds of rare events by pricing out-of-the-money options. They just respond to supply and demand. The notion of "implied probability distribution" is merely a Dutch-book compatibility type of proposition.

Bachelier-Thorp

We conclude with the following remark. Sadly, all the equations, from the first (Bachelier), to the last pre-Black-Scholes-Merton (Thorp) accommodate a scale-free distribution. The notion of explicitly removing the expectation from the forward was presented in Keynes (1924) and later by Blau (1944) –and long a Call short a put of the same strike equals a forward. These arbitrage relationships appeared to be well known in 1904.

This is why we call the equation Bachelier-Thorp. We were using it all along and gave it the wrong name, after the wrong method and with attribution to the wrong persons. It does not mean that dynamic hedging is out of the question; it is just not a central part of the pricing paradigm. It led to the writing down of a certain stochastic process that may have its uses, some day, should markets "spiral towards dynamic completeness". But not in the present.

[7]See Gârleanu, Pedersen, and Poteshman (2006).

References

Bachelier, L. (1900): Theory of speculation in: P. Cootner, ed., 1964, The random character of stock market prices, MIT Press, Cambridge, Mass.

Bernhard, A. (1970): More Profit and Less Risk: Convertible Securities and Warrants. Written and Edited by the Publisher and Editors of The Value Line Convertible Survey, Arnold Bernhard & Co., Inc

Black, F., and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81, 637–654.

Blau, G. (1944-1945): "Some Aspects of The Theory of Futures Trading," The Review of Economic Studies, 12(1).

Boness, A. (1964): "Elements of a Theory of Stock-Option Value," Journal of Political Economy, 72, 163–175.

Bouchaud J.-P. and M. Potters, 2003, *Theory of Financial Risks and Derivatives Pricing, From Statistical Physics to Risk Management*, 2nd Ed., Cambridge University Press.

Bronzin, V. (1908): Theorie der Prämiengeschäfte. Leipzig und Wien: Verlag Franz Deticke.

Cootner, P. H. (1964): The Random Character of Stock Market Prices. Cambridge, Massachusetts, The M.I.T. Press.

De La Vega, J. (1688): Confusión de Confusiones. Re-printed in the book: 'Extraordinary Popular Delusions and the Madness of Crowds & Confusión de Confusiones' edited by Martin S. Fridson, 1996, New York: Wiley Publishing.

Derman, Emanuel, and Iraj Kani, 1994, Riding on a smile, Risk 7, 32–39.

Derman, Emanuel, 1998, Stochastic implied trees: Arbitrage pricing with stochastic term and strike structure of volatility, International Journal of Theoretical and Applied Finance 1, 61–110.

Derman, E., and N. Taleb (2005): "The Illusion of Dynamic Delta Replication," Quantitative Finance, 5(4), 323–326.

Dupire, Bruno, 1994, Pricing with a smile, Risk 7, 18–20.

Dupire, Bruno, 1998, A new approach for understanding the. impact of volatility on option prices, Discussion paper Nikko Financial Products.

Dupire, Bruno, 2005, Volatility derivatives modeling,

www.math.nyu.edu/ carrp/mfseminar/bruno.ppt.

Filer, H. 1959: Understanding Put and Call Option's, New York: Popular Library.

Gabaix, X. , P. Gopikrishnan, V.Plerou & H.E. Stanley,2003, A theory of power-law distributions in financial market fluctuations, Nature, 423, 267-270.

Gann, W. D. (1937) How to Make Profits in Puts and Calls.

Gârleanu, N., L. H. Pedersen, and Poteshman (2006): "Demand-Based Option Pricing". Working Paper: New York University - Leonard N. Stern School of Business.

Gatheral, Jim (2006), The volatility Surface, Wiley

Haug E. G. (1996): The Complete Guide to Option Pricing Formulas, New York: McGraw-Hill

Haug, E. G. (2007): Derivatives Models on Modes, New York, John Wiley & Sons

Higgins, L. R. (1902): The Put-and-Cal l. London: E. Wilson.

Keynes, J. M. (1924): A Tract on Monetary Reform. Reprinted 2000, Amherst New York: Prometheus Books.

Mandelbrot,B. 1997, Fractals and Scaling in Finance, Springer-Verlag.

Mandelbrot, B. 2001a,*Quantitative Finance*, 1, 113-123

Mandelbrot,B. 2001b,*Quantitative Finance*, 1, 124-130

Mandelbrot,B., 1963, The variation of certain speculative prices. The Journal of Business, 36(4):394–419.

Mandelbrot and Taleb (2007), "Mild vs. Wild Randomness: Focusing on Risks that Matter." Forthcoming in Frank Diebold, Neil Doherty, and Richard Herring, eds., *The Known, the Unknown and the Unknowable in Financial Institutions*. Princeton, N.J.: Princeton University Press.

Merton R. C. (1973): "Theory of Rational Option Pricing," Bell Journal of Economics and Management Science, 4, 141–183.

Merton R. C. (1976): "Option Pricing When Underlying Stock Returns are Discontinuous," *Journal of Financial Economics*, **3**, 125–144.

Merton, RC (1992): *Continuous-Time Finance*, revised edition, Blackwell

Nelson, S. A. (1904): The A B C of Options and Arbitrage. New York: The Wall Street Library.

Reinach, A. M. (1961): The Nature of Puts & Calls. New York: The Book-mailer.

Review of the International Statistics Institute, 37(3).

Rubinstein M. (1998): Derivatives. www.in-the-money.com

Rubinstein M. (2006): A History of The Theory of Investments. New York: John Wiley & Sons.

Ruffino and Treussard (2006) "Derman and Taleb's `The Ilusion of Dynamic Replication': a comment", Quantitative Finance, 6, 5, pp 365-367

Samuelson, P. (1965): "Rational Theory of Warrant Pricing," Industrial Management Review, 6, 13–31.

Sprenkle, C. (1961): "Warrant Prices as Indicators of Expectations and Preferences," Yale Economics Essays, 1(2), 178–231.

Stanley, H.E. , L.A.N. Amaral, P. Gopikrishnan, and V. Plerou, 2000, Scale invariance and universality of economic fluctuations", Physica A, 283,31-41

Stoll, H. (1969): "The Relationship Between Put And Call Prices," Journal of Finance, 24(5), 801–824.

Taleb, N. (1997): Dynamic Hedging. New York: John Wiley & Sons.

Taleb, N. (2007): The Black Swan. New York: Random House.

Taleb. N. (2007): "Scale-Invariance in Practice: Some Questions and Workable Patches"

Thorp, E. O. (2002): "What I Knew and When I Knew It – Part 1, Part 2, Part 3," Wilmott Magazine, Sep-02, Dec-02, Jan-03.

Thorp, E. O. (1969): "Optimal Gambling Systems for Favorable Games," Thorp, E. O., and S. T. Kassouf (1967): Beat the Market. New York: Random House.

Thorp, E. O. (2007): "Edward Thorp on Gambling and Trading," in Haug (2007).